Japanese-European Symposium on Symplectic Varieties and Moduli Spaces – Sixth Edition

Schedule Bologna

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30-10:00	Fu I	Fu II	M. Kapustka (online) starts at 9	Yoshioka I	Yoshioka II
10:00-10:45	**Joint question time and discussion**				
10:45-11:15	**Coffee break**				
11:15-12:15	Amerik (online)	Oberdieck (r)	Video Shimada (r)	Brakkee (r)	
12:30-14:30	**Lunch and free discussion time**				
14:30-15:30	Video Mukai	Video Hatano	Free discussion time	Video Odaka	
15:30-16:00	**Coffee break**				
16:00-17:00	Barros (r)	Grossi (r)	Song (r)	Sarti (r)	
17:15-18:15			Video Okumura	Video Sano	

Schedule Tokyo

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00			Video Oberdieck		
10:15-11:15		Video Amerik	Shimada (r)	Odaka (r)	
11:30-12:30		Hatano (r)	Video Grossi	Video Song	Video Brakkee
12:15-14:30		**Lunch and free discussion time**			
14:30-15:00	**Registration**				
15:00-16:00	Mukai (r)	Video Barros	Okumura (r)	Sano (r)	Video Sarti
16:00-16:30	**Coffee break**				
16:30-18:00	Fu I	Fu II	M. Kapustka starts at 17	Yoshioka I	Yoshioka II
18:00-18:45	**Joint question time and discussion**				

Short courses

Lie Fu (Radboud University)

Title Franchetta property for K3 surfaces and hyper-Kähler varieties.

Abstract The Franchetta conjecture (now a theorem) says that the universal family over the moduli space of genus $g > 1$ curves has relative Picard group generated by the relative canonical class. Such a phenomenon that the generic fiber of certain universal family has "small" Chow groups generated by "canonical" elements is referred to as the Franchetta property. I will give an overview of a series of work carried out in recent years in collaboration with R. Laterveer and C. Vial on the Franchetta property for universal K3 surfaces (conjectured by O'Grady) and hyper-Kähler varieties.

Kota Yoshioka (Kobe University)

Title Moduli of stable sheaves on an abelian surface

Abstract Moduli of stable sheaves on an abelian surface has a symplectic structure and an irreducible symplectic manifold of the generalized Kummer type appeas as a Bogomolov factor. For the study of these spaces, Fourier-Mukai transforms and Bridgeland stability play important roles. In my talk, I first explain Fourier-Mukai transforms and Bridgeland stability on an abelian surface. Then I explain their applications to the study of moduli of stable sheaves.

Abstracts

Ekaterina Amerik (HSE and Orsay)

Title Parabolic automorphisms of hyperkähler manifolds

Abstract As observed by Serge Cantat, an automorphism of a K3 surface which is a fiberwise translation in an elliptic pencil, acts with dense orbits in the fibers; as a consequence, if a K3 surface has two such automorphisms (with distinct pencils), then the group they generate acts ergodically. We show that the same holds for arbitrary hyperkähler manifolds, as an application of Deligne semisimplicity theorem. This is a joint work with Misha Verbitsky.

Ignacio Barros Reyes (University Paris-Saclay)

Title Kodaira classification of the moduli of hyperelliptic curves

Abstract We study the birational geometry of the moduli spaces of hyperelliptic curves with marked points. We show that these moduli spaces have non Q-factorial singularities forcing us to work with a better birational model provided by Hurwitz spaces. We finish the Kodaira classification proving that these spaces are of intermediate type when the number of markings is $4g+6$ and of general type when the number of markings $n \geq 4g+7$. Similarly we consider a finite cover given by marking the Weierstrass points. In this case we show that the Kodaira dimension is one when $n = 4$ and of general type when $n \geq 5$. Further, we provide a full classification of the structure of the pseudo-effective cone. We show the cone is non-polyhedral when the number of markings is at least two and polyhedral in the remaining cases. This is joint work with Scott Mullane.

Emma Brakkee (University of Amsterdam)

Title General type results for moduli of IHS varieties

Abstract In 2007, Gritsenko, Hulek and Sankaran proved that the moduli space of K3 surfaces of degree 2d is of general type when $d > 61$. Their strategy is to reduce the question to the existence of a certain cusp form for an orthogonal modular variety. This method has been applied successfully to prove general type results for, among others, some moduli of higher-dimensional IHS varieties. In this talk, we sketch the reduction argument and give general type results for two more types of IHS moduli spaces. We also explain what the challenges are when trying to imitate the strategy for other moduli spaces of IHS varieties. This is joint work in progress with I. Barros, P. Beri and L. Flapan.

Annalisa Grossi (Chemnitz University of Technology)

Title Ihs varieties as symplectic quotients of ihs manifolds

Abstract We aim at constructing ihs varieties with trivial algebraic regular fundamental group starting from an ihs manifold X and a finite group G of symplectic actions on X. I will present some work in progress in the case where X is a generalized Kummer fourfold or an O'Grady's sixfold. The case of generalized Kummer fourfolds is the content of a joint work with Bertini, Capasso, Mauri and Mazzon. Moreover I will present some classification results about symplectic birational transformations of manifolds of OG6 type (joint work with Onorati and Veniani) and how these results could be exploit for our purpose.

Kohei Hatano (Hokkaido University)

Title Cohomology of conical symplectic resolutions

Abstract In this talk, I will introduce some examples of conical symplectic resolutions and symplectic dual. In 2015, Hikita conjectured that the cohomology of the conical symplectic resolution is isomorphic to the coordinate ring of the $\mathbb{C}^*$- fixed points of the affinization of the symplectic dual. Finally, I will explain Hikita's conjecture in the case of the framed moduli space.

Michal Kapustka (IMPAN Krakow)

Title Relations between EPW sextics and EPW cubes

Abstract Double EPW cubes and double EPW sextics are among few known locally complete families of hyperkaehler manifolds. They both arise from the same data: a Lagrangian space in the third wedge power of a six-dimensional space equipped with the natural symplectic form given by wedge product. In this talk we consider relations between these two families. In particular, we compare their period maps and investigate moduli spaces of minimal degree elliptic curves on very general manifolds in both families. As a consequence, combining our results with known results on double EPW sextics, we prove that a very general double EPW cube is the moduli space of stable

objects on the Kuznetsov component of its corresponding Gushel-Mukai fourfolds; this answers a problem posed by Perry, Pertusi and Zhao. We moreover show that the moduli spaces of minimal degree elliptic curves on a very general double EPW sextic and the corresponding double EPW cube are isomorphic curves. This provides further evidence for a conjecture of Nesterov and Oberdieck and raises interesting questions. This is joint work with Grzegorz Kapustka and Giovanni Mongardi.

Shigeru Mukai (RIMS Kyoto)

Title The Vinberg-Conway chain terminates just after a supersingular symplectic 6-fold with 100 (-2)-divisors.

Abstract Leech roots calculate fundamental domains of the hyperbolic lattices $U + D_n$ up to $n = 22$. Generalizing a result of Vinberg (1983, $n = 18$), we apply this calculation for $n < 21$ to (semi-)automorphism groups of various K3 surfaces, and realize the case $n = 21$ as the Picard lattice of the symplectic manifold in characteristic 2 in the title, which is expected to have an action of the Higman-Sims simple group.

Georg Oberdieck (HCM Bonn)

Title Enumerative Geometry of symplectic 4-folds

Abstract I will talk about joint work with Yalong Cao and Yukinobu Toda in which we study the Donaldson-Thomas theory of 1-dimensional sheaves on symplectic 4-folds. The invariant count (virtually) 1-dimensional stable sheaves on the 4-fold. Our main conjecture relates these counts via Gopakumar-Vafa theory to Gromov-Witten invariants. We prove the conjecture for the product of two K3 surfaces, where it naturally leads to an interesting formula for the Fujiki constants of the Chern classes of the tangent bundle of the Hilbert schemes of points of a K3 surfaces. In $K3^{[2]}$-type the conjecture leads to explicit predictions for the number of isolated genus two curves of minimal degree on a very general symplectic 4-folds of $K3^{[2]}$-type.

Yuji Odaka (Kyoto University)

Title Collapsing hyperKahler metrics and moduli compactifications for K3 surfaces, II

Abstract This is a sequel to my talk at the 2nd of this nice workshop series, held at Levico Terme in 2017. The theme is to interrelate the moduli compactifications of K3 surfaces, hyperKahler manifolds and the limiting behaviour of hyperKahler metrics in differential geometry, to provide applications to both sides. Last time I mainly focused on the case of type III degenerations (after reviewing the cases of curves and abelian varieties). This year I plan to focus on some progress on type II degenerations, and explain that some PL convex functions on the interval arise as a new invariant (joint with Y.Oshima). If time permits, we explain recent related results of Alexeev-Brunyate-Engel and Alexeev-Engel, to which we also give fairly different proofs.

Katsuhiko Okumura (Waseda University)

Title SNC log symplectic structures on the blow-up of projective spaces

Abstract A SNC log symplectic structure is a generically symplectic Poisson structure with a reduced and simple normal crossing (SNC) degeneracy divisor. The typical examples are the diagonal Poisson structures on a projective space. In 2014, Lima-Pereira showed that SNC log symplectic structures on the Fano variety of Picard rank one must be the diagonal Poisson structures on projective space. This theorem can be generalized to the product of the Fano variety (O 2020). However, when we assume that the variety is Fano variety, no examples are known except the diagonal Poisson structure on a (product of) projective space so far. In this talk, we will classify and construct SNC log symplectic structures on the blow-up of projective spaces along a linear subspace. We will find two new examples of SNC log symplectic structures, which are NOT induced by the diagonal Poisson structure as a result.

Taro Sano (Kobe University)

Title Construction of non-Kähler Calabi-Yau manifolds by log deformations

Abstract It is an open problem whether there exist only finitely many topological types of (strict) projective Calabi-Yau manifolds in a fixed dimension. Clemens, Friedman and Reid constructed infinitely many topological types of non-Kähler Calabi-Yau 3-folds by conifold transitions. In this talk, I will explain how to construct infinitely many topological types of non-Kähler Calabi-Yau manifolds in any dimension by log deformations of SNC varieties.

Alessandra Sarti (University of Poitiers)

Title Complex Reflection Groups and K3 surfaces

Abstract We classify all K3 surfaces that one can obtain as quotient of surfaces in complex projective three space by certain subgroups of finite complex reflection groups of rank four, the K3 surfaces that we obtain have at worst ADE singularities. The proof of the classification avoid as much as possible a case-by-case analysis and involves the theory of finite complex reflection groups, in particular Lehrer-Springer theory. We obtain several families and we show that each family contains K3 surfaces with the maximum Picard number, which is 20. This construction generalizes and explains in some sense a result of 2003 by W. Barth and myself. The results presented in the talk come from joint papers with C. Bonnafé.

Ichiro Shimada (Hiroshima University)

Title Automorphism groups of Enriques surfaces (joint work with Simon Brandhorst)

Abstract We explain a method to calculate the automorphism group and the nef cone of an Enriques surface by means of the geometry of polytopes in hyperbolic space.

Jieao Song (University of Paris-Diderot)

Title Topological constraints of hyperkähler manifolds

Abstract We study some topological constraints for hyperkähler manifolds. I will present two results: a description of their cobordism classes in terms of known examples (joint with Georg Oberdieck and Claire Voisin), and a conditional bound on the second Betti number (joint with Thorsten Beckmann). I will also talk about some conjectural properties on the positiveness of certain numerical invariants called generalized Fujiki constants, and explain the consequences.